Do you know?

HCF is also known as GCD which means Greatest Common Divisor.

### (b) Prime Factorisation Method



So, HCF of 144 and 204 is 12.

Now, let us find the HCF of 12 and the third number, i.e. 252.

HCF = 12 
$$\longrightarrow$$
 12 )252 (21  
 $252$   
0 Remainder = 0  
Division stops

So, the HCF of 204, 144 and 252 = 12.

**Example 5:** Find the greatest number that will divide 140, 170, 155 leaving remainder 5 in each case.

Solution: Here, we have to find a number which exactly divides (140 - 5), (170 - 5), (155 - 5)

The required number is the HCF of 135, 165 and 150.

First take any two numbers, say 135 and 165.



10

15)150

150 0

HCF of 165 and 135 is 15.

Now, we find the HCF of 15 and 150

The required number is 15.

- **Example 6:** The floor of a room is 6 m 75 cm long and 5 m wide. It is to be paved with square tiles. Find the largest size of tile needed.
- **Solution:** In order to find the largest size of tile needed, we find the number that divides 675 and 500 exactly.



35

The largest size of tile needed is 25 cm.

### Worksheet 5

- 1. Find the HCF of the following numbers by factor method.
  - (a) 7, 18 (b) 12, 30, 54 (c) 70, 14, 35
- 2. Find the HCF of the following numbers by prime factorisation method.
  - (a) 76, 28 (b) 24, 16, 36 (c) 38, 64, 82
- 3. Find the HCF of the following numbers by continued division method.
  - (a) 345, 506 (b) 144, 384, 120
  - (c) 287, 533 (d) 208, 494, 949
  - (e) 1212, 6868, 1111 (f) 1794, 2346, 4761
  - (g) 70, 105, 175 (h) 270, 450, 315

### 4. What is the HCF of-

- (a) two consecutive natural numbers.
- (b) two consecutive even numbers.
- (c) two consecutive odd numbers.
- (d) any two prime numbers.
- 5. Find the greatest number which divides 203 and 434 leaving remainder 5 in each case.
- 6. Find the greatest number which will divide 625 and 1433 leaving remainders 5 and 3 respectively.
- 7. The length, breadth and height of a room are 8.25 m, 6.75 m and 4.50 m respectively. Determine the longest tape which can measure the three dimensions of the room exactly.
- There are 312 mango bites, 260 eclairs and 156 coffee bites in a box. These are to be put in packets so that each packet contains the same number of toffees. Find the maximum number of toffees in each packet.

### LEAST COMMON MULTIPLE (LCM)

LCM of two or more numbers is the **Least Common Multiple** of these numbers. Let us find the LCM of 3, 6 and 9.

### (a) By Listing Multiples



The least common multiple of these three numbers is 18.

So, LCM of 3, 6, 9 is 18.

### (b) Prime Factorisation Method

Let us find the LCM of 24, 15 and 45.



### (c) Common Division Method

Let us find the LCM of 30, 45, 60.



### Let us solve some examples.

- **Example 7:** Find the smallest number which when divided by 25, 40, 60 leaves remainder 7 in each case.
- Solution: The required number will be 7 added to the least common multiple (LCM) of these numbers.

Let us first find the LCM.

Therefore, the required number = 600 + 7 = 607. Let us check.



Example 8: In a morning walk, three boys step off together. Their steps measure 80 cm, 85 cm and 90 cm respectively. What minimum distance should each walk so that all can cover the distance in complete steps?

Solution: The minimum distance needed will be the Least Common Multiple (LCM) of 80, 85, 90.

2	80, 85, 90
2	40, 85, 45
2	20, 85, 45
2	10, 85, 45
3	5, 85, 45
3	5, 85, 15
5	5, 85, 5
17	1, 17, 1
	1, 1, 1

 $LCM = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 17$ 

Required distance = 12240 cm

or 122 m 40 cm

Worksheet 6

- 1. Find LCM of the following numbers by listing their multiples.
  - (a) 12, 9 (b) 4, 5, 2 (c) 25, 15
- 2. Find the LCM by prime factorisation method.
  - (a) 10, 15, 6 (b) 16, 12, 18 (c) 25, 30, 40
- 3. Find the LCM by common division method.

(a) 12, 15, 45	(b) 24, 90, 48
(c) 30, 24, 36, 16	(d) 16, 48, 64
(e) 35, 49, 91	(f) 20, 25, 30
(g) 12, 16, 24, 36	(h) 40, 48, 45

- 4. Find the least number which when divided by 40, 50 and 60 leaves remainder 5 in each case.
- 5. Three Haryana Roadways buses stop after 50, 100 and 125 km respectively. If they leave together, then after how many kilometres will they stop together?

6. Four bells toll at intervals of 8, 9, 12 and 15 minutes respectively. If they toll together at 3 p.m., when will they toll together next?

### **PROPERTIES OF HCF AND LCM**

- **1.** HCF of given numbers is not greater than any of the numbers.
  - e.g. HCF of 5 and 15 = 5

HCF of 12 and 18 = 6

- 2. LCM of given numbers is not smaller than any of the numbers.
  - e.g. LCM of 5, 15 = 15

LCM of 12, 18 = 36

- **3.** HCF of given numbers is a factor of their LCM.
  - e.g. HCF of 16, 12 = 4 LCM of 16, 12 = 48 HCF 4 is a factor of LCM 48
- 4. LCM of given numbers is a multiple of their HCF.

e.g. HCF of 16, 12 = 4

LCM of 16, 12 = 48

LCM 48 is a multiple of HCF 4

5. If HCF of two numbers is one of the number then LCM is the greater number.

e.g. HCF of 5 and 15 = 5

LCM = 15 (greater number)

6. HCF of co-prime numbers is 1.

5 and 9 are co-prime

HCF = 1

7. LCM of co-prime numbers is the product of the numbers.

LCM of 5 and 9 = 45

8. Product of HCF and LCM of two numbers is equal to the product of the numbers.

HCF of 9 and 12 = 3LCM of 9 and 12 = 36Product of 9 and 12 = 108Product of HCF and LCM =  $3 \times 36 = 108$ 

### Let us study some examples.

**Example 9:** Find HCF and LCM of 25, 65.

**Solution:** Here, we find only HCF of 25 and 65.



LCM will be found by using the property.

Product of numbers = Product of HCF and LCM.

$$25 \times 65 = 5 \times \text{LCM}$$
  
LCM =  $\frac{25^{5} \times 65}{5_{1}} = 325$ 

**Example 10:** HCF of two numbers is 16 and their product is 6400. Find their LCM.

Solution: We have,

HCF × LCM = Product of numbers 16 × LCM = 6400 LCM =  $\frac{6400^{400}}{16_1}$  = 400 LCM = 400

### Worksheet 7

- 1. For each of the following pairs of numbers, verify that product of numbers is equal to the product of their HCF and LCM.
  - (a) 10, 15 (b) 35, 40 (c) 32, 48
- 2. Find HCF and LCM by using the property in Question no. 1.
  - (a) 27, 90 (b) 145, 232 (c) 117, 221
- 3. Can two numbers have 16 as HCF and 380 as LCM? Give reasons.

- 4. The HCF of two numbers is 16 and product of numbers is 3072. Find their LCM.
- 5. The LCM and HCF of two numbers are 180 and 6 respectively. If one of the number is 30, find the other.
- 6. LCM of two numbers 160 and 352 is 1760. Find their HCF.
- 7. Write 'True' or 'False' for the following statements.

(a)	LCM of two numbers is a factor of their HCF.	
(b)	Product of three numbers is equal to the product of their HCF and LCM.	
(c)	HCF of given numbers is always a factor of their LCM.	
(d)	LCM of given numbers cannot be smaller than the numbers.	
(e)	LCM of co-prime numbers is equal to their product.	

# ALUE BASED QUESTIONS

1. The schools nowadays have a council which consists of student representatives. This council helps the school in organising various events. Rohan has also been selected in his school council this year.

The school organised a picnic in which 108, 162 and 270 students of Classes-VI, VII and VIII respectively were going. Rohan's teacher asked him to help the transport incharge.

- (a) Find out the number of buses required, if each bus had to carry maximum but equal number of students from each class.
- (b) As a member of school council Rohan was made a part of decision making. What other values does the student council develop in a child?
- (c) Suggest one way by which you can help your school if you are selected as a council member.
- 2. You know 5 June is celebrated every year as World Environment Day. As a part of its celebration, Vrinda and her two friends decided to have a cycling race to promote environmental friendly transport. They started at 12 noon and took 3 minutes 20 seconds, 3 minutes 40 seconds and 4 minutes respectively to cycle on a circular track.



- (a) If Vrinda and her friends started together at 12 noon, then when will they meet next?
- (b) Suggest the ways by which you can save the environment.

# **BRAIN TEASERS**

1.

A. Tick ( $\checkmark$ ) the correct answer.

- (a) Six bells commence tolling together and toll at intervals of 2, 4, 6, 8, 10 and 12 seconds respectively. After how many minutes will they toll together again? (i) 5 minutes (ii) 6 minutes (iii) 4 minutes (iv) 2 minutes The HCF of two numbers is 11 and their LCM is 7700. If one of the numbers is (b) 275, then the other number is-(i) 279 (ii) 283 (iii) 308 (iv) 318 The greatest possible length which can be used to measure exactly the lengths (C) 7 m, 3 m 85 cm, 12 m 95 cm is-(iv) 42 cm (i) 35 cm (ii) 25 cm (iii) 15 cm (d) 252 can be expressed as a product of primes as-(i)  $2 \times 2 \times 3 \times 3 \times 7$ (ii)  $2 \times 2 \times 2 \times 3 \times 7$ (iii)  $3 \times 3 \times 3 \times 3 \times 7$ (iv)  $2 \times 3 \times 3 \times 3 \times 7$ Which of the following is a factor of every natural number? (e) (ii) 0 (iii) – 1 (i) 1 (iv) any number B. Answer the following questions. Find the highest common factor of 36 and 84. (a) (b) How many factors does 36 have? Express 132 as the sum of two odd primes. (C) (d) What should be added to 4057 to make it divisible by 9? (e) Find the HCF of 95, 105 and 115 by continued division. 2. Are 32 and 34 co-prime numbers? Why? 3. Write any four twin primes between 50 and 110. 4. Express the greatest 3-digit number as a product of primes.
  - 43

- 5. Express the smallest 5-digit number as a product of primes.
- 6. State which of the following numbers are divisible by both 3 and 9?

(a) 235674 (b) 78015

- 7. Test which of the following numbers are divisible by 11.
  - (a) 147246 (b) 2352825
- 8. What least number should be subtracted from the following numbers to make them divisible by 3?
  - (a) 2825 (b) 856291
- 9. What least number should be added to the following numbers to make them divisible by 9?
  - (a) 42724 (b) 39065
- 10. Replace the blank in 625 with the least number, so that the number is divisible by 11.
- 11. Write any two numbers which are-
  - (a) divisible by 3 but not 9.
  - (b) divisible by 5 but not 10.
  - (c) divisible by both 4 and 8.
  - (d) divisible by 2, 4 and 8.
- 12. Find the HCF of 1624, 522 and 1276.
- 13. Find the LCM of 198, 135, 108 and 54.
- 14. The HCF and LCM of two numbers are 13 and 1989 respectively. If one number is 117, find the other.
- 15. Can two numbers have 15 as HCF and 350 as LCM? Why?

## HOTS

Find the greatest number of four digits which is divisible by 15, 20 and 25.



# **ENRICHMENT QUESTION**

# To find the factors of a number, you have to find all the pairs of numbers that multiply together to give that number.

The factors of 48 are:

1 and 48 2 and 24 3 and 16 4 and 12 6 and 8

If we leave out the number we started with, 48, and add all the other factors, we get 76:

1 + 2 + 3 + 4 + 6 + 8 + 12 + 16 + 24 = 76

So ... 48 is called an **abundant number** because it is less than the sum of its factors (without itself). (48 is less than 76.)

A number less than the sum of its factors except itself is called an **abundant number**.

See if you can find some more abundant numbers!

# YOU MUST KNOW

- 1. Two prime numbers whose difference is 2 are called twin prime numbers.
- 2. Two numbers are said to be co-prime when they have only 1 as common factor.
- 3. Every number has infinite number of multiples and finite number of factors.
- 4. A number is divisible by another number if it is divisible by its co-prime factors.
- 5. If a number is divisible by another number, then it is divisible by each factor of that number.
- 6. If a number is divisible by two co-prime numbers, then it is divisible by their product.
- 7. If two given number are divisible by a number, then their sum is also divisible by that number.
- 8. If two given number are divisible by a number, then their difference is also divisible by that number.
- 9. Prime factorisation of a number is the factorisation in which every factor is a prime number.
- 10. HCF is also known as the Greatest Common Divisior (GCD).
- 11. Product of HCF and LCM of two numbers is equal to the product of the numbers.
- 12. HCF of given numbers is not greater than any of the numbers.
- 13. LCM of given numbers is not smaller than any of the numbers.





INTRODUCTION NEED FOR INTEGERS

Observe the number line drawn below.



**INTEGERS** 

On the number line, 0 (zero) is the starting point (called the **origin**) and all the natural numbers are to the right of 0.



Now let us consider some situations.

Situation 1: See! these two cats have pounced on a piece of cake that was on the plate.



Let us take the plate as the starting point 0. We have two numbers on the opposite sides of 0.

3 m to the **right** of 0 and 2 m to the **left** of 0.

Situation 2: See! Rohan and Sohan are going to a shop to make some purchases.



Now, let us take the ground level as origin 0. Here, we have 2 numbers on the opposite sides of 0.

16 steps **above** 0 and 9 steps **below** 0.



In the above two situations,

3 m to the right of 0 is represented as	+ 3
2 m to the left of 0 is represented as	- 2
climbing up 16 steps is represented as	+ 16
climbing down 9 steps is represented as	- 9
Similarly,	
A profit of ₹ 200 is	+ 200
8°C below the freezing point is	- 8
Depositing ₹ 500 in a bank is	+ 500

Numbers with positive (+) sign are called **positive numbers**.

Numbers with negative (-) sign are called negative numbers.



Negative integers  $-1, -2, -3, \dots$  are read as minus one, minus two, minus three, etc.

## **OPPOSITES**

- Opposite of the **PROFIT** of ₹ 20 is **LOSS** of ₹ 20.
- Opposite of 5°C **ABOVE** freezing point is 5°C **BELOW** freezing point.
- Opposite of -3 is +3.

### Worksheet 1

- 1. Indicate the following by using integers.
  - (a) Earning ₹ 500
  - (b) Loss of ₹ 90
  - (c) Climbing up 10 steps
  - (d) Withdrawal of ₹ 500 from a bank
  - (e) 5 m above sea level
  - (f) 3 km towards north
  - (g) 10°C below zero
  - (h) An increase of 25 marks

### 2. Write the opposites of-

- (a) Depositing ₹ 1,000 in a bank account.
- (b) Decrease of 5 marks.
- (c) Earning ₹ 200.
- (d) Going 2 km towards east.
- (e) Two steps to the left of zero on a number line.
- (f) Losing weight of 7 kg.
- 3. Encircle the negative integers from the following numbers.

-59, 6, 0, -1, -4, 45, -62, 107

### REPRESENTATION OF INTEGERS ON A NUMBER LINE

We know that negative integers are opposite of positive integers. So let us mark the negative integers on the left of zero on the number line.



So, now we have the number line ...







### Note:

- Every positive integer is greater than any negative integer.
- Zero is less than every positive integer.
- Zero is greater than every negative integer.
- - 1 is the greatest negative integer.
- We cannot find the greatest positive integer or the smallest negative integer.

## **ABSOLUTE VALUE OF INTEGERS**



See! Sonu and Monu are standing at a point zero (0). After two minutes, see their position.



Here the distance walked by both of them is same (3 units) without taking into account the direction (sign). So, we can say that the absolute value of 3 and -3 is **3**.

### Let us find the absolute value of some integers.

	Integer	Absolute Value	
The absolute	+ 1	1	
value of an	- 1	1	The absolute
integer is greater than	- 7	7	value of any non-zero intege
or equal to the	21	21	is positive.
integer.	0	0	

The symbol used to write absolute value is **two vertical lines (**||**)**, one on either side of the integer. Thus, the absolute value of -7 is written as |-7| = 7

### Worksheet 2

- 1. Do as directed.
  - (a) Mark any point as origin on the given number line.
  - (b) Write integers on either side of the origin with proper signs.

- 2. Encircle the number which is to the right of the other on number line in each of the following pairs.
  - (a) 3, -1(b) 0, -8(c) -6, -4(d) 14, -7(e) -9, -8(f) 4, 7
- 3. Write all the integers between-
  - (a) -5 and 0 (b) -4 and 3 (c) -11 and 1 (d) -6 and -1
- 4. Compare the numbers and insert an appropriate symbol (>, <, =) in the given space.
  - (a) 3 3 (b) -1 0 

     (c) -101 104 (d) -82 28 

     (e) -4 14 (f) 16 16 

     (g) -97 98 (h) -197 96 

     (i) 0 7 (j) -1 1 

     52

5. Fill in the following table with the absolute values.

Integer	Absolute value
17	
- 18	
0	
- 43	
21	
- 105	
- 61	
1283	

- 6. Write the following in ascending order.
  - (a) 4, -5, 16, -11, -21, 50
  - (b) 0, -1, 7, -16, -12, -30
- 7. Write the following in descending order.
  - (a) 171, 26, 43, 103, 105, 77
  - (b) 9, -8, 0, -75, -79, 93
- 8. Write 'True' or 'False' for the following statements.
  - (a) Every integer is either positive or negative.
  - (b) Zero is greater than every negative integer.
  - (c) An integer to the left of another integer is always smaller.
  - (d) We can find the smallest integer.
  - (e) Absolute value of a given integer is always greater than the integer.
  - (f) All natural numbers are positive integers.
  - (g) All whole numbers are integers.
  - (h) Absolute value of 3 is -3.

# **OPERATIONS ON INTEGERS**

### A. ADDITION OF INTEGERS

Let us find the position of following numbers on number line.

(a) 3 more than -1 (b) 4 less than -2

(a) 3 more than -1

### Step II



So, the number 3 more than -1 is 2.

(b) 4 less than -2



So, the number 4 less than -2 is -6.

To find a number more than a given number, we proceed to the right and to find a number less than a given number, we go to the left.

Now, let us perform the operation of addition on the number line.



(i) Addition of two positive integers

Add (+ 3) and (+ 2) on a number line.



prefix the sign of addends.

To add two positive integers or two negative integers, add their absolute values and prefix the sign of addends to the sum.

### (iii) Adding one positive and one negative integer



We can also do this sum without the help of number line.

$$|-4| = 4$$

$$|9| = 9$$
We take the absolute values.  

$$= + (9-4)$$
We find the difference of absolute values.  

$$= + 5$$
We prefix the sign of the integer whose  
absolute value is greater.

If integers have opposite signs, we find the difference of their absolute values and prefix the sign of the integer whose absolute value is greater.

### Worksheet 3

### 1. Use the number line and write the number which is:

- (a) 3 more than 4
- (c) 7 more than 8
- (e) 5 more than 6
- 2. Find the sum on a number line.
  - (a) 8 + (- 3)
  - (c) (-5) + (-4)
  - (e) 7 + (- 4) + (- 3)

(f) 7 less than 0

(b) 5 less than 1

(d) 2 less than 2

- (b) 7 + 2
- (d) (-2) + 1 + (-2)
- (f) (-2) + (-3) + (-4)

### 3. Add the following:

(a)	67, – 49	(b)	– 452, 138
(c)	– 95, – 35	(d)	6951, - 6952
(e)	1001, - 101	(f)	- 381, - 619
(g)	- 419, 386, 419	(h)	- 19, 158, - 103
(i)	- 9005, 360	(j)	- 65, - 35, 100

### **PROPERTIES OF ADDITION**

**Property-1:** The sum of any two integers is also an integer.

Let us add + 5 and - 9 + 5 + (-9) = -4  $\longrightarrow$  (-4) is also an integer. Integers

Property-2: The sum remains the same even if we change the order of the addends.

Consider the sum of -6 and +11



Property-3: Sum of three integers remains the same even after changing the grouping of the addends.

Now, add 3, -5, 9 [3 + (-5)] + 9] First we add 3 and (-5) = (-2) + 9 We add the sum to 9 = 7Now, let us change the groupings. 3 + [(-5) + 9] Grouping is changed = 3 + 4 We add the sum to 3 = 7See! Sum remains the same.

Property-4: When zero is added to any integer, the sum is the integer itself.

We have, 3 + 0 = 0 + 3 = 3

-11 + 0 = 0 + (-11) = -11

*Note:* **Zero** is called the identity element for addition.

Property-5: When one is added to any integer, we get its successor.

Let us see what will happen when we add one to any integer.

10 + 1 = + 11 + 11 is the successor of + 10

-7 + 1 = -6 -6 is the successor of -7

Property-6: Every integer has an additive inverse such that their sum (integer and additive inverse) is equal to zero.

Consider the following sums.

5 + (-5) = 0	<ul> <li>5 is the opposite of 5</li> </ul>
-8+8=0	8 is the opposite of $-8$

The opposite of an integer is also called the **negative** or **additive inverse** of the integer.

Worksheet 4

1. Find the sum in two different ways.

(a)	- 32, 50	(b)	– 81, – 79
(C)	64, - 100	(d)	13, - 78, 15

2. Write the additive inverse of the following:

(a)	31	(b)	- 7
(c)	21	(d)	- 501
(e)	0	(f)	- 34

- 3. Find the sum using the properties of addition.
  - (a) 200 + (-105) + (-36)
  - (b) (-45) + 100 + (-55)
  - (c) (-825) + 725 + 100 + (-100)
  - (d) 927 + (-517) + (-518)

(e) (-215) + (-215) + 860 + (-215) + (-215) + 1

- 305 + (-5) + (-2) + 2 + (-200)(f)
- (g) 637 + 350 + (-237) + (-900)
- (h) (-99) + 7 + (-101) + 93

#### 4. Fill in the following blanks.

- The negative of 3 is (a)
- (b) (-8) + = (-8)
- (c) 11 + (-16) = + 11
- (d) [(-3) + 5] + 6 = (-3) + [+ 1
- The identity element of addition is (e)
- (-51) + 51 =(f)

#### 5. Write 'True' or 'False' for the following statements.

- (a) 3 + (-5) is not an integer.
- Sum of two negative integers is also a negative integer. (b)
- (c) Negative of 15 does not exist.
- (d) [(-3) + 8] + (-4) = [8 + (-3)] + (-4)
- (e) 91 + (-41) = (-91) + 41
- (f) -46 + 0 = 0
- (g) Sum of a positive integer and a negative integer is always negative.
- (h) |-9-5| = |-9| |5|

### **B. SUBTRACTION OF INTEGERS**

lf









Let us do more examples.



To subtract two integers, we add the negative of the subtrahend to the minuend.

### **PROPERTIES OF SUBTRACTION**

**Property-1:** The difference of any two integers is also an integer.

e.g. 3 - (+5) = -2- 2 is an integer.

Property-2: Every integer has its predecessor.

e.g. the predecessor of -5 is (-5) - 1 = -6

**Property-3:** Zero subtracted from any integer is the integer itself.

e.q. - 6 - 0 = -6

### Worksheet 5

- Write the negative of the following integers. 1.
  - (a) 3 (c) 100 (b) 5 (d) - 91 (e) 108 (f) - 2004
- Fill in the following blanks. The first one is done for you. 2.



- 3.
  - (a) 9, 4 (b) - 9, 4 (d) - 11, - 6 (c) 10, -7(e) 16, 0 (f) 2001, 201 (g) 458, - 263 (h) 0, - 565 (i) - 823, - 232 41623, 26413 (i)
- Subtract 6 from 3 and 3 from 6. Are the results same? 4.
- Sum of two integers is 48. If one of them is -25, find the other. 5.
- Subtract the sum of 38 and 49 from 100. 6.

### 7. Compare.

(a) 
$$(-25) - (-15)$$
  $(-25) + (-15)$   
(b)  $18 + (-8)$   $18 - (-8)$ 

- 8. Find the value of-
  - (a) (- 3) (- 19)
  - (c) 56 (- 13) + 15
  - (e) (-16) (-6) + (-9) 4
- (b) -12 8 (-35)(d) (-41) + (-36) - 23
- (f) 71 83 (- 42) + 15

### **C. MULTIPLICATION OF INTEGERS**

- (i) Multiplication of two positive integers
  - Let us multiply + 3 by + 4
  - (+ 3)  $\times$  (+ 4) means + 4 is added 3 times
  - (+ 4) + (+ 4) + (+ 4) = + 12





When both integers are positive, we multiply their absolute values and prefix plus sign to the product.

(ii) Multiplication of a positive and a negative integer

Let us multiply  $(+2) \times (-4)$  - 4 is repeatedly added two times

 $(+2) \times (-4) = (-4) + (-4) = -8$ 

We take two jumps of 4 units each to the left.



We reach at – 8

So,  $(+2) \times (-4) = -8$ 

When one integer is positive and the other is negative, we multiply their absolute values and prefix minus sign to their product.

### (iii) Multiplication of two negative integers

See the following pattern



When both integers are negative, we multiply their absolute values and prefix plus sign.

*Note:* The teacher should take a few more examples to show the pattern.

### (iv) Product of more than three factors

Find the product of  $(-2) \times 3 \times (-1) \times 5 \times (-5)$ 

$$= (-6) \times (-1) \times 5 \times (-5)$$
  
=  $6 \times 5 \times (-5)$   
=  $30 \times (-5)$   
=  $-150$ 

In multiplication, if the number of negative integers is-

- odd, the product is **negative.**
- even, the product is **positive.**

### **PROPERTIES OF MULTIPLICATION**

### **Property-1:** Product of any two integers is also an integer.

We have,  $(-2) \times (+4) = -8$  (-8 is also an integer)

**Property-2:** Product remains the same even if we change the order of integers.



Property-3: Product remains the same even when we change the groupings of the integers.

Let us multiply  $[2 \times (-10)] \times 3$  in two different ways.



**Property-4:** Product of an integer and zero is zero.

We have,

$$(-5) \times 0 = 0$$
  
 $(+19) \times 0 = 0$ 

Property-5: 1 multiplied by any integer is the integer itself.

We have,  $(-9) \times 1 = -9$  $(+24) \times 1 = +24$ 

*Note:* One (1) is the identity element of multiplication.

**Property-6:** This property is called the distributive property of multiplication over addition.

If 2, (-3), 5 are three integers then,  $2 \times [(-3) + 5] = 2 \times (-3) + 2 \times 5$ We have  $2 \times [(-3) + 5] = 2 \times (-3) + 2 \times 5$   $= 2 \times 2 = (-6) + 10 = 4$ Same 64

### Worksheet 6



- (a) 7 negative and 3 positive integers.
- (b) 26 negative and 10 positive integers.

- (c) 11 negative and 11 positive integers.
- (d)  $(-4) \times (-5) \times (-6) \times \_\_\_ \times (-13).$
- (e)  $(-12) \times (-13) \times (-14) \times (-15) \times \_\_\_ \times (-22).$

### 7. Write 'True' or 'Flase' for the following statements.

- (a) The product of two integers is always an integer.
- (b) The product of two integers with opposite signs is positive.
- (c) The identity element of multiplication is 0.
- (d) Of the two integers if one is negative, the product must be negative.

### **D. DIVISION OF INTEGERS**

We know that every multiplication fact has two corresponding division facts.



### (i) Division of integers with like signs.



To divide two integers of like signs, we divide their absolute values and prefix plus (+) sign.

_			
_			
-			i.

### (ii) Division of integers with unlike signs

Divide 6 by (-3)  $(+6) \div (-3) = (-2)$  negative sign opposite sign Now, divide 75 by (-15)  $(+75) \div (-15) = (-5)$ opposite sign opposite sign

To divide two integers of opposite signs, we divide their absolute values and prefix minus (–) sign.

### **PROPERTIES OF DIVISION**

**Property-1:** The quotient of two integers is not always an integer.

We have,  $6 \div (-2) = -3$  (-3 is an integer) Is  $2 \div (-3)$  an integer? Is  $(-6) \div 4$  an integer?

**Property-2:** When an integer (non-zero) is divided by the same integer, the quotient is one.

We have,  $(-3) \div (-3) = 1$   $(+10) \div (+10) = 1$ 

- Property-3: When an integer is divided by one, the quotient is the same integer. We have,  $(-7) \div 1 = -7$   $(+3) \div 1 = +3$
- Property-4: Zero divided by any integer (non-zero) is zero.

We have,  $0 \div (-9) = 0$   $0 \div (+3) = 0$ 

Worksheet 7

1. Put the appropriate sign in the quotients.



### 2. Find the quotient of the following:

- (a) (- 36) ÷ 9
- (c) (- 5375) ÷ (- 25)
- (e) (- 108) ÷ 12
- (g)  $(-3000) \div 100$
- (i) 48 ÷ (- 16)
- 3. Fill in the following blanks.
  - (a)  $(-93) \div = (-93)$ (c)  $\div (-8) = 0$
  - (e) (- 65) ÷ \_\_\_\_ = 1

- (b) 125 ÷ (- 5)
  - (d)  $374 \div (-17)$
  - (f)  $0 \div (-17)$
  - (h)  $(-144) \div (-12)$
  - (j)  $(-1331) \div (-11)$
- (b)  $17 \div (-1) =$ (d)  $\div 1 = -42$

### POWER OF INTEGERS

Remember Area of a square = Side × Side Volume of a cube = Edge × Edge × Edge

Now, let us look at the area of this square.

Area of square =  $4 \text{ cm} \times 4 \text{ cm} = 16 \text{ cm}^2$  $4 \times 4 \text{ can also be written as } 4^2$ So,  $4^2 = 4 \times 4$ 

Now, look at the volume of this cube.

Volume of this cube =  $5 \times 5 \times 5 = 125$  cm<sup>3</sup>

$$5 \times 5 \times 5$$
 can also be written as  $5^3$ 

$$5^3 = 5 \times 5 \times 5$$



or







5 cm



### In 2<sup>4</sup>, 2 is called the Base and 4 is called the Power or Exponent



We write	We read	
2 <sup>2</sup>	Two square or two to the power two	
6 <sup>3</sup>	Six cube or six to the power three	
(- 7) <sup>4</sup>	Minus seven to the power four	

### See these examples.



2. Complete the table given below. The first one is done for you.

Pow	vered number	Base	Exponent
(a)	7 <sup>5</sup>	7	5
(b)	9 <sup>3</sup>		
(c)	(- 3)4		
(d)	(- 1) <sup>6</sup>		
(e)	20 <sup>2</sup>		
(f)	(- 10) <sup>7</sup>		

- 3. Write in power notation.
  - (a) 4 × 4 × 4
  - (b)  $(-2) \times (-2) \times (-2) \times (-2)$
  - (c)  $5 \times 5 \times 5 \times 5 \times 5$
  - (d)  $(-10) \times (-10) \times (-10) \times \dots 8$  times
  - (e)  $(-11) \times (-11) \times (-11)$
  - (f)  $(-1) \times (-1) \times (-1)$  ......... 33 times

### 4. Write the following in expanded form.

- (a)  $2^5$  (b)  $3^4$
- (c)  $(-7)^3$  (d)  $(-12)^2$
- 5. Compute the following:
  - (a)  $3^4$  (b)  $(-5)^2$
  - (c)  $(-1)^{78}$  (d)  $11^3$
  - (e)  $(-4)^3 \times (-10)^3 \times (-1)^{789}$

### 6. Find the number which is—

- (a) Cube of 9
- (c) 5th power of (-10)

### 7. Simplify.

- (a)  $3^2 + 4^2$
- (c)  $1^3 + 2^3 + 3^3$
- (e)  $3^3 (-2)^3$

(b) Square of 15

 $(50)^2$ 

(f)

- (d) 19th power of (-1)
- (b)  $2^3 4^2$
- (d)  $(-10)^3 + (-10)^2 + (-10)^1$
- (f)  $(-1)^{16} + (-1)^{36} + (-1)^7 + (-1)^{54}$
- 70

	8.	Subtract the cube of (– 2) from the cube of 2.						
	9.	Verify.						
		(a)	$(-2)^5 \times (-2)^3 = (-2)^8$	(b)	$6^5 \times 6^4 = 6^9$			
		(c)	$5^2 - 3^2 = 4^2$	(d)	$12^2 + 5^2 = 13^2$			
	10.	0. Write 'True' or 'False' for the following statements.						
		(a)	$3^4 = 4^3$					
		(b)	$9^7 \div 9^5 = 9^2$					
	(c) $(-5)^2 \times (-5)^3 \times (-5) = (-5)^6$							
		(d)	$6^3 + 6^2 = 6^{3+2}$					
		(e) Cube of a negative integer is positive.						
		(f)	$(-1)^{101} = -1$					
		(g)	1 <sup>3</sup> = 3					
		(h)	Cube of a positive integer is neg	ative				
		(i)	$3^2 = 6$					
<ul><li>(j) 6th power of a negative integer is positive.</li><li><b>11. What power of</b>—</li></ul>								
		(a)	2 is 32	(b)	– 4 is – 64			

(c) 10 is 100000 (d) (-5) is - 125

# ALUE BASED QUESTIONS

- Ravi and Rahul were good friends. Ravi was a poor boy. He was very much in need of a geometry box. Rahul decided to help him. He bought for him a geometry box costing ₹ 65 from his pocket money. Ravi was very excited to get the new geometry box and thanked Rahul for his caring nature.
  - (a) Express spending ₹ 65 as an integer.
  - (b) Suggest any two ways by which you have helped any of your friends.
- 2. In a quiz competition there were 25 questions. 2 marks was alotted to every correct answer and -1 to every wrong answer. Sheetal attempted 22 questions out of which 2 answers were wrong. The teacher gave her 40 marks. Sheetal went to the teacher and

informed her that she has been given more marks. The teacher was happy with Sheetal. She did not deduct her marks.

- (a) What is Sheetal's actual score?
- (b) What quality of Sheetal made the teacher happy?

# **BRAIN TEASERS**

1. A. Tick ( $\checkmark$ ) the correct answer. (a) The number of integers between (- 10) and 3, is-(i) 11 (ii) 13 (iii) 12 (iv) 14 (b) If we subtract (-10) from (-11) we get-(i) – 1 (iii) – 21 (ii) 1 (iv) 21 Square of 2 subtracted from cube of (-1) is-(C) (i) 3 (iii) – 5 (ii) 5 (iv) 1 (d) Value of |-7| + (-6) + |3| is-(i) - 10 (ii) 4 (iii) 10 (iv) -4(e) Which of the following does not lie to the right side of (-61) on the number line? (iii) – 49 (i) - 10 (ii) 18 (iv) - 73 B. Answer the following questions. Write any two integers less than (-101). (a) Find the value of |(-30) - (-7)|. (b) Which integer added to (-4) will give the integer 5? (C) Simplify and write its opposite  $(-3) \times 5 \times (-1)$ . (d) Find the sum of the greatest negative integer and smallest positive integer. (e) 2. Indicate using integers. (b) 5° Celsius below zero 200 BC (a) Win by 3 goals (d) 40 km above sea level (C) 3. Write the opposites of the following statements. (a) India won the match by 3 wickets.

(b) Mohan withdrew ₹ 2500 from his bank account. 4. Write any three integers which are— (a) smaller than -25(b) greater than – 191 5. Arrange in ascending order. - 104, 48, - 69, 13, - 7, - 96, - 48, 5 6. Find the value on the number line. (b) 8 + (-6) + (-2)(a) (-3) + 5 - 77. Simplify. (a) (-400) + 781 + (-1400) + (-81) + 300(b) (-273) + (-541) + 900 + (-511)8. Subtract. (b) - 115 from - 115 (a) -9 from 0 9. Find the value of-(a)  $(-6) \times [9 + (-11)]$ (b)  $325 \times (-641) + 325 \times (-359)$ (c)  $5^2 \times (-1)^{19} \times (-2)^3 \times 3^2 \times (-10)^3$ 10. Compare.  $18 \times (-3) + 21$  and  $18 \times [(-3) + 21]$ 11. Fill in the blanks. (a) There are \_\_\_\_\_ integers from - 4 to 11? (b) Natural numbers are called \_\_\_\_\_ integers (positive/negative). The additive inverse of 6 is \_\_\_\_\_ (C) (d) 168 + \_\_\_\_ = 0 The predecessor of – 249 is (e)  $[(-2) + (-7)] \times 3 = 3 \times \_\_\_+3 \times \_\_\_$ (f) The opposite of  $(-3) \times 2 \times (-1)$  is \_\_\_\_\_ (g) All the negative integers are \_\_\_\_\_ than zero. (h) 14, 7, 0, – 7, \_\_\_\_\_, \_\_\_\_\_. (i)

### 12. Write 'True' or 'False' for the following statements.

- The absolute value of an integer is always greater than the integer. (a)
- (b) The product of 9 negative integers is positive.
- (C) Cube of 11 has 1 in its units place.
- The base in  $7^3$  is 3. (d)
- $3^8 \div 3^5 = 3^3$ (e)
- 13. Fill in the missing places with proper integers.
  - (a)



# HOTS

- 1. (a) Calculate  $1 2 + 3 4 + 5 6 + \dots + 179 180$ .
  - (b) Find the value of  $5 + (-5) + 5 + (-5) + 5 + \dots$  if the number of fives are-
    - (i) 148
    - (ii) 191
- 2. A cement company gains ₹ 12 per bag of white cement sold and gets a loss of ₹ 8 per bag of grey cement sold.
  - (a) If the company sells 3500 bags of white cement and 5000 bags of grey cement in a month, find the gain or loss.
  - (b) If the number of grey cement bags sold is 6000, how many bags of white cement should the company sell to have neither gain or loss?





# YOU MUST KNOW

- 1. We need to use numbers with negative signs in some situations. These are called negative numbers. Some examples of their use are temperature of a day, water level in a sea, etc.
- 2. Positive numbers, negative numbers along with zero are called integers. Zero is neither positive or negative.
- 3. Each and every integer can be represented on the number line. The integer to the right side of another integer is greater.
- 4. Absolute value of an integer is the numerical value without taking the sign to account.
- 5. To add two positive integers or two negative integers, add their absolute values and prefix the sign of addends to it.
- 6. If integers are of opposite signs, we find the difference of their absolute values and prefix the sign of the integer whose absolute value is greater.
- 7. To subtract two integers, we add the negative of the subtrahend to the minuend.
- 8. In multiplication, if both integers have like signs we multiply their absolute values and prefix plus sign to the product and if the integer have unlike signs we multiply their absolute values and prefix negative sign to the product.
- 9. One is the identity element of multiplication of integers.
- 10. To divide two integers of like signs, we divide their absolute values and prefix (+) sign.
- 11. To divide two integers of unlike signs, we divide their absolute values and prefix (-) sign.
- 12. In 7<sup>3</sup>, 7 is called the base and 3 is called exponent or power. Power or exponent indicates the number of times the base is to be multiplied by itself.