CLASS - XII

SUBJECT- MATHEMATICS

INVERSE TRIGONOMETRIC FUNCTIONS

HOLIDAYS H.W.

SESSION 2018-19

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. Write the principal value of

(i) $\sin^{-1}\left(-\sqrt{3/2}\right)$ **(ii)** $\cos^{-1}(\sqrt{3}/2)$

(iii) $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$ **(iv)** $\text{cosec}^{-1}(-2)$

(v) $\cot^{-1}\left(\frac{1}{\sqrt{3}}\right)$ **(vi)** $\sec^{-1}(-2)$.

(vii) $\sin^{-1}\left(\frac{-\sqrt{3}}{2}\right) + \cos^{-1}\left(\frac{-1}{2}\right) + \tan^{-1}(-1/\sqrt{3})$

2. What is the value of the following functions (using principal value)

(i) $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) - \sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$ **(ii)** $\sin^{-1}\left(-\frac{1}{2}\right) - \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

(iii) $\tan^{-1}(1) - \cot^{-1}(-1)$ **(iv)** $\text{cosec}^{-1}(\sqrt{2}) + \sec^{-1}(\sqrt{2})$

(v) $\tan^{-1}(1) + \cot^{-1}(1) + \sin^{-1}(1)$. **(vi)** $\sin^{-1}\left(\sin\frac{4\pi}{5}\right)$

(vii) $\tan^{-1}\left(\tan\frac{5\pi}{6}\right)$ **(viii)** $\text{cosec}^{-1}\left(\text{cosec}\frac{3\pi}{4}\right)$

(ix) $\cos\left\{\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right) + \frac{\pi}{6}\right\}$

3. If $\tan^{-1}x + \tan^{-1}y = \frac{4\pi}{5}$, find $\cot^{-1}x + \cot^{-1}y$.

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

4. Show that: $\tan^{-1}\left[\frac{\sqrt{1+\cos x}+\sqrt{1-\cos x}}{\sqrt{1+\cos x}-\sqrt{1-\cos x}}\right] = \frac{\pi}{4}+\frac{x}{2}$; $x \in [0,\pi]$

5. Prove that :

$$\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right) - \cot^{-1}\left(\sqrt{\frac{1+\cos x}{1-\cos x}}\right) = \frac{\pi}{4} \quad x \in (0, \pi/2)$$

6. Prove that $\tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right) = \sin^{-1}\frac{x}{a} = \cos^{-1}\left(\frac{\sqrt{a^2-x^2}}{a}\right)$.

7. prove that:

$$\cot^{-1}\left[2\tan\left(\cos^{-1}\frac{8}{17}\right)\right] + \tan^{-1}\left[2\tan\left(\sin^{-1}\frac{8}{17}\right)\right] = \tan^{-1}\left(\frac{300}{161}\right)$$

8. Prove that:

$$\tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right) = \frac{\pi}{4}+\frac{1}{2}\cos^{-1} x^2$$

9. Solve:

$$\cot^{-1} 2x + \cot^{-1} 3x = \frac{\pi}{4}$$

10. Prove that:

$$\tan^{-1}\left(\frac{m}{n}\right) - \tan^{-1}\left(\frac{m-n}{m+n}\right) = \frac{\pi}{4}, m, n > 0$$

11. Prove that:

$$\tan\left[\frac{\pi}{4}+\frac{1}{2}\tan^{-1}\left(\frac{a}{b}\right)\right] + \tan\left[\frac{\pi}{4}-\frac{1}{2}\tan^{-1}\left(\frac{a}{b}\right)\right] = \frac{2\sqrt{a^2+b^2}}{b}$$

Relations and Functions

- Binary operation * defined on set A is called associative iff $a * (b * c) = (a * b) * c \ \forall\ a, b, c \in A$
- If * is Binary operation on A, then an element $e \in A$ (if exists) is said to be the identity element iff $a * e = e * a = a\ \forall a \in A$
- Identity element is unique.
- If * is Binary operation on set A, then an element $b \in A$(if exists) is said to be inverse of $a \in A$ iff $a * b = b * a = e$
- Inverse of an element, if it exists, is unique.

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. If A is the set of students of a school then write, which of following relations are Universal, Empty or neither of the two.

   $R_1 = \{(a, b) : a, b$ are ages of students and $|a - b| > 0\}$

   $R_2 = \{(a, b) : a, b$ are weights of students, and $|a - b| < 0\}$

   $= \{(a, b) : a, b$ are students studying in same class$\}$

2. Is the relation R in the set $A = \{1, 2, 3, 4, 5\}$ defined as

   $R = \{(a, b) : b = a + 1\}$ reflexive?

3. If R, is a relation in set N given by

   $R = \{(a, b) : a = b - 3, b > 5\}$,

   then does element $(5, 7) \in R$?

4. If $f : \{1, 3\} \to \{1, 2, 5\}$ and $g : \{1, 2, 5\} \to \{1, 2, 3, 4\}$ be given by $f = \{(1, 2), (3, 5)\}$, $g = \{(1, 3), (2, 3), (5, 1)\}$, write gof.

5. Let $g, f : R \to R$ be defined by

$g(x) = \frac{x+2}{3}$, $f(x) = 3x - 2$. write fog (x)

6. If $f : R \to R$ defined by

$f(x) = \frac{2x-1}{5}$

be an invertible function, write $f^{-1}(x)$.

7. If $f(x) = \log x$ and $g(x) = e^x$. Find fog and gof, $x > 0$.

8. Let * be a Binary operation defined on R, then if

(i) $a * b = a + b + ab$, write $3 * 2$

(ii) $a * b = \frac{(a+b)^2}{3}$, write $(2*3)*4$.

9. If $n(A) = n(B) = 3$, then how many bijective functions from A to B can be formed?

10. If $f(x) = x + 1$, $g(x) = x - 1$, then (gof) (3) = ?

11. Is $f : N \to N$ given by $f(x) = x^2$ one-one? Give reason.

12. If $f : R \to A$, given by

$f(x) = x^2 - 2x + 2$ is onto function, find set A.

13. If $f : A \to B$ is bijective function such that $n(A) = 10$, then $n(B) = ?$

14. If $f : R \to R$ defined by $f(x) = \frac{x-1}{2}$, find $(fof)(x)$

15. $R = \{(a, b) : a, b \in N, a \neq b \text{ and } a \text{ divides } b\}$. Is R reflexive? Give reason

16. Is $f : R \to R$, given by $f(x) = |x - 1|$ one-one? Give reason

17. $f : R \to B$ given by $f(x) = \sin x$ is onto function, then write set B.

Pg 5

18. If $f(x) = \log\left(\frac{1+x}{1-x}\right)$, show that $f\left(\frac{2x}{1+x^2}\right) = 2f(x)$.

19. If '*' is a binary operation on set Q of rational numbers given by $a * b = \frac{ab}{5}$ then write the identity element in Q.

20. If * is Binary operation on N defined by $a * b = a + ab \;\forall\; a, b \in N$, write the identity element in N if it exists.

SHORT ANSWER TYPE QUESTIONS (4 Marks)

21. Check the following functions for one-one and onto.

(a) $f : R \to R$, $f(x) = \frac{2x-3}{7}$

(b) $f : R \to R$, $f(x) = |x + 1|$

(c) $f : R - \{2\} \to R$, $f(x) = \frac{3x-1}{x-2}$

(d) $f : R \to [-1, 1]$, $f(x) = \sin^2 x$

22. Consider the binary operation * on the set {1, 2, 3, 4, 5} defined by $a * b$ = H.C.F. of a and b. Write the operation table for the operation *.

23. Let $f : R - \left\{\frac{-4}{3}\right\} \to R - \left\{\frac{4}{3}\right\}$ be a function given by $f(x) = \frac{4x}{3x+4}$

Show that f is invertible with $f^{-1}(x) = \frac{4x}{4-3x}$

24. Let R be the relation on set $A = \{x : x \in Z, 0 \leq x \leq 10\}$ given by $R = \{(a, b) : (a - b)$ is divisible by $4\}$. Show that R is an equivalence relation. Also, write all elements related to 4.

25. Show that function $f : A \to B$ defined as $f(x) = \frac{3x+4}{5x-7}$ where $A = R - \left\{\frac{7}{5}\right\}$, $B = R - \left\{\frac{3}{5}\right\}$ is invertible and hence find f^{-1}.

26. Let * be a binary operation on Q such that $a * b = a + b - ab$.

(i) Prove that * is commutative and associative.

(ii) Find identify element of * in Q (if it exists).

~ copies