Relations and Functions

1. Show that the relation R in the set A = { $x \in Z : 0 \le x \le 12$}, given by R = {(a,b) : |a-b| is a multiple of 4} is an equivalence relation. [4]

2. If $f(x) = \frac{4x+3}{6x-4}$, $x \neq 2/3$, show that fof(x) = x, for all $x \neq 2/3$. What is the inverse of f. [4]

3. Let * be a binary operation on N defined by a*b = HCF of a and b .Is * commutative and associative. Does there exist identity element for this binary operation. [4]

4. Let f: R→R be defined as f(x) = 10x + 7. Find the function g : R→R such that gof= fog=I_R [4]

5. For binary operation *defined on Z^+ as a*b = 2^{ab}. Determine whether * is commutative or associative. [4]

6. Let f: N→N given by $f(n) = \begin{cases} \frac{n+1}{2}, & n\, odd \\ \frac{n}{2}, & n\, even \end{cases}$

   For all $n \in N$ .Check whether the function f is bijective. [4]

7. Find the number of binary operation on the set {a,b}. [1]

8. Let f : R→R given by f(x) =$(3- x^3)^{1/3}$ then find f0f(x). [1]

9. If f(x) = $8x^3$ and g(x) = $x^{1/3}$, find gof. [1]

10. Prove that the function f : R→R , given by f(x) = 2x , is one-one. [1]

APPLICATION OF DERIVATIVES

(1) Find the point at the point on the curve at which the tangent $x^2+y^2=1$ parallel to y axis.
(2) Find equation of tangent $y= x^2$ at (0,0).
(3) Verify mean value theorem for the function $f(x) = x^2$ in the interval [2,4]
(4) A square piece of tin of side 18 cm is to be made into a box without top,by cutting a square from each corner and folding up the flaps to form the box. What should be the side of the square to be cut off so that the volume of the box is maximum possible.
(5) Verify Rolles theorem for the function $f(x)= x^2+2x-8$, x Є[-4,2]
(6) Find the approximate change in the volume V of acube of side x meters caused by increasing the side by 1%.
(7) Show that of all the rectangle inscribed oin affixed circle the square has the maximum area.
(8) Find the interval of the $f(x) = x^2(x-2)^2$ is a increasing function.
(9) A particle moves along the curve $6y =x^3+2$.Find the point on the curve at which the y coordinate is changing 8 times as fast as x coordinate.
(10)Find the equation of tangent to the given curve $y= 3x^2-3x$.

CHAPTER-MATRICES

General Instructions:

(i) Q.No.1-3 carry 1 mark each.
(ii) Q.no. 4-8 carry 4 marks each.
(iii) Q.No.9-10 carry 6 marks each

Q.1 If a matrix has 24 elements;what is the possible order it can have?

Q.2 If A= $\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ and I= $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, Find k so that A^2= kA-2I.

Q.3 If A and B are symmetric matrices of same order , then write whether AB-BA is symmetric or skew-symmetric matrix.

Q.4 If 2X+3Y = $\begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$, and 3X+5Y= $\begin{bmatrix} 1 & -2 \\ -6 & 5 \end{bmatrix}$;Then find X and Y.

Q.5 Given $3\begin{bmatrix} x & y \\ z & w \end{bmatrix}=\begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix}+\begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix}$, Find the values of x,y,z and w.

Q.6 Find : x,y ,z if A= $\begin{bmatrix} 0 & 2y & z \\ z & y & -z \\ x & -y & z \end{bmatrix}$ satisfies $A' = A^{-1}$.

Q.7 Express the following matrix as a sum of symmetric and skew-symmetric matrix.

$$A = \begin{bmatrix} 2 & 4 & 6 \\ 7 & 3 & 5 \\ 1 & -2 & 4 \end{bmatrix}$$

Q.8 Show that $\begin{bmatrix} 1 & -\tan\frac{\theta}{2} \\ \tan\frac{\theta}{2} & 1 \end{bmatrix}\begin{bmatrix} 1 & \tan\frac{\theta}{2} \\ -\tan\frac{\theta}{2} & 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$.

Q.9 Using elementary transformation,find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -1 \\ 2 & 1 & 6 \end{bmatrix}$$

Q.10 If A= $\begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & -1 \\ 1 & 2 & 3 \end{bmatrix}$, Then show that $A^3-4A^2-3A+11I=0$.Hence find A^{-1}.

Vectors

Q.1 – Simplify $[\vec{a} - \vec{b}\ ,\ \vec{b} - \vec{c}\ ,\ \vec{c} - \vec{a}\ ]$ (4)

Q.2 – The position vectors of A , B , C and D are 3i – 2j – k , 2i + 3j – 4k , -i + j + 2k and 4i + 5j + λ k respectively. If the points A , B , C , D lie in a plane i.e. they are co planers , find the value of λ. (4)

Q.3 - Find the value of p for which the vectors $\vec{a}$ = 2i + 3j – k and $\vec{b}$ = 4i + 6j + pk are (a) parallel (b) perpendicular (1)

Q.4- If $\vec{a}$, $\vec{b}$, $\vec{c}$ are position vectors of the vertices A , B , and C of a triangle ABC , show that

Ar. Δ ABC = $\frac{1}{2}\left|\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}\right|$ (4)

Q.5 – Find the magnitude of two vectors $\vec{a}$ and $\vec{b}$ having the same magnitude and the angle between them is 60^{o} and their scalar product is $\frac{1}{2}$. (4)

Q.6 - If the vertices A , B , C of Δ ABC are (1,2,3) , (-1,0,0) , (0,1,2) respectively. Then find $\angle$ABC. (4)

Q.7 – Show that the vectors 2i – j + k , i – 3j – 5k and 3i – 4j – 4k form the vertices of a right angled triangle. (4)

Q.8 - Find a unit vector $\perp$ to each of vectors ($\vec{a}+\vec{b}$) and ($\vec{a}$ - $\vec{b}$) where $\vec{a}$ = 3i + 2j + 2k and $\vec{b}$ = i + 2j – 2k
(4)

Q.9 – If a unit vector $\vec{a}$, makes angles $\frac{\pi}{3}$ with I , $\frac{\pi}{4}$ with j and an acute angle θ with K then find θ and hence the components of $\vec{a}$. (4)

Q.10 – Show that each of the given three vectors is a unit vector $\frac{1}{7}$ [2i + 3j + 6k] , $\frac{1}{7}$ [3i – 6j + 2k] , $\frac{1}{7}$ [6i + 2j – 3k] . Also show that they are mutually $\perp$ to each other (4)

CONTINUITY AND DIFFERENTIABILITY

1. Differentiate: $e^{m\tan^{-1}x}$ with respect to x. 1
2. Differentiate: $\sec(\tan\sqrt{x})$ with respect to x. 1
3. Find the value of 'k' so that the function $f(x)=\begin{cases}\dfrac{k\cos x}{\pi-2x}; & x\neq \pi/2\\ 3; & x=\pi/2\end{cases}$ is continuous at $x=\pi/2$ 4
4. Find the derivative of the function: $\left(x+1/x\right)^{x}+x^{\left(1+\frac{1}{x}\right)}$ with respect to x 4
5. Differentiate the function $y=x^{\sin x-\cos x}+\dfrac{x^2-1}{x^2+1}$ 4
6. If $x=a(\cos t+\log(\tan t/2)$ and $y=a\sin t$, find $\frac{dy}{dx}$. 4
7. If $e^{m\tan^{-1}x}$, prove that $(1-x^2)\dfrac{d^2y}{dx^2}-x\dfrac{dy}{dx}=m^2y$ 4

8. If $x = a(\cos t + t\sin t)$ and $y = a(\sin t - t\cos t)$, find $\frac{d^2y}{dx^2}$ 4

9. If $x = \sin(\frac{1}{a}\log y)$, show that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = a^2 y$ 4

10. If $x = \frac{\sin^3 t}{\sqrt{\cos 2t}}$ and $y = \frac{\cos^3 t}{\sqrt{\cos 2t}}$, find $\frac{dy}{dx}$ 4

INTEGRALS

Evaluate:

Q1. $\int dx/(4+3\cos x)$

Q2. $\int \sqrt{x^2 + 2x + 5}\, dx$

Q3. $\int_0^{\pi} (x+a)/(x-a)\, dx$

Q4. $\int \mathrm{Cosec}^3 x\, dx$

Q5. $\int (x^2+1)/(x^4 + x^2 +3)dx$

6. $\int_0^{\infty} \frac{\tan^{-1} x dx}{(1+x^2)^{3/2}}$.

7. $\int_0^{2\pi} |Cosx| dx$

8. $\int dx/(\sin x + \sqrt{3}\cos x)$

9. $\int_0^2 e^x$ Sinx dx

10. One kind of cake requires 400g of flour and 50g of fat and another kind of cake requires 200g of flour and 100g of fat. Find the maximum number of cakes which can be made from 10Kg flour and 2Kg of fat assuming that there is no shortage of other ingredients used in making the cakes.

11. Evaluate : $\int_0^1 \frac{dx}{1+x^2}$. (1)

12. Evaluate : $\int \frac{\sin^{-1} x}{\sqrt{1-x^2}}\, dx.$ (1)

13. Evaluate : $\int \frac{\sec^2 x}{1+\tan^2 x}\, dx$ (1)

14. Evaluate : $\int \frac{x \sin^{-1} x}{\sqrt{1-x^2}}\, dx$ (4)

15. Evaluate : $\int \frac{1}{\sin(x-a)\cos(x-b)}\, dx$ (4)

16. Evaluate : $\int_0^{\pi} \frac{x}{1+\cos^2 x}\, dx.$ (4)

17. Evaluate : $\int_0^4 (|x| + |x-3|)dx.$ (4)

18. Evaluate as a limit of sums : $\int_0^4 (x^2 + e^{3x})dx.$ (6)

19. Evaluate : $\int \frac{\sin x}{(2-\cos x)(1-\cos x)}\, dx.$ (6)

20. Evaluate : $\int (5x+3)\sqrt{x^2+4x+10}\, dx.$ (6)

APPLICATION OF INTEGRALS

1.Find the area enclosed by the curve y=sin x and the lines x=0,x=∏ and the x-axis.

2.Find the area of the region bounded by the parabola $y=x^2$ and y=IxI.

3.Find the area of the region bounded by the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$.

4.Find the area of the region enclosed between the two circles $x^2+y^2=4$ and $(x-2)^2+y^2=4$.

5.Find the area lying above x-axis and included between the circle $x^2+y^2=8x$ and the parabola $y^2=4x$.

6. Find the area of the region $\{(x,y);y^2\leq 4x,4x^2+4y^2\leq 9\}$.

7. Find the area enclosed by the curve y=cos x and the lines x=0 and x=2∏.

8.Find the area of the parabola $y^2=4ax$ bounded by its latus rectum.

9.Find the area of the region bounded by the curves $y=x^2+2$,y=x,x=0 and x=2.

10. Using integration find the area of the triangle ABC, coordinates of whose vertices are A(2,0),B(4,5) and C(6,3).

1.	Find $\sin^{-1}(\sin 10)$	1

2.	Find x if $Sin\left[\cos^{-1}\frac{2\sqrt{6}}{5}+\cos^{-1}x\right] = 1$	1
3.	Find Cot^{-1}[2 cos (2 cosec^{-1} 2)]	1
4.	Find the value of $\tan^{-1}\left(\tan\frac{5\pi}{6}\right)$	1
5.	If $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \pi/2$	4
6.	$\Pr ove\ that\ \tan^{-1}\left[\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right]$	4
7.	Find the value of $\tan^{-1}1 + \tan^{-1}2 + \tan^{-1}3$	4
8.	$If\ \cos^{-1}\frac{x}{3}+\cos^{-1}\frac{y}{4}=\alpha, then\ prove\ that\ \frac{x^2}{9}+\frac{y^2}{16}-\frac{xy\cos\alpha}{6}=\sin^2\alpha$	4
9.	Solve : $\tan^{-1}3x + \tan^{-1}4x = \frac{\pi}{4}$	4
10.	Simplify : $\tan^{-1}\left[\frac{2\cos x-3\sin x}{3\cos x+2\sin x}\right]\ if\ \frac{2}{3}\tan x > -1$	4

Differential Equations

1. Write the order and degree of following differential equation 1
   $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^4 + sin\left(\frac{dy}{dx}\right) = 0$
2. Write the number of arbitrary constants in the general solution of :
   $\frac{dy}{dx} + \left(\frac{d^2y}{dx^2}\right)^2 + 5y = 0$ 1
3. Form the differential equation for family of circles having centre on x-axis and radius 2 units. 4
4. Find the equation of the curve passing through $(2\ ,\ 1)$ having differential equation $xdy - ydx = 0$ 4
5. Solve : $\frac{dy}{dx} = \frac{1}{x+y+1}$ 4
6. Solve : $\left(\frac{dy}{dx} - \frac{y}{x}\right)\sin\frac{y}{x} + 1 = 0 \quad , \quad y = 0\ when\ x = 1$ 4
7. Solve : $\frac{dy}{dx} = \frac{(x+2y)}{(x-y)}$ 6
8. Solve : $(1-y^2)\frac{dx}{dy} + (x-a)y = 0 \quad , \quad -1 < y < 1$ 6
9. Solve : $\sqrt{1+x^2+y^2+x^2y^2} + \frac{dy}{dx} = 0$ 6

10. In a culture, the bacteria count is 1, 00,000. The number is increased by 10% in two hours. In how many hours will the count reach 2, 00,000, if the rate of growth of bacteria is proportional to the number present? 6

3-Dimensional geometry.

PATTERN: Q1 TO Q 4 (1 MARKS),Q5 TO Q7(4 MARKS) AND Q8 TO Q10 (6 MARKS).

1. Find the equation of the line passing through the point (2,1,3) having the direction ratios 1,1,-2.
2. Write the vector equation of the following line $\frac{x-5}{3} = \frac{y+4}{3} = \frac{z-5}{-2}$.
3. Write the distance of the following planes from the origin 2x-y+2z+1=0.
4. Cartesian of the line AB is $\frac{2x-1}{2} = \frac{4-y}{7} = \frac{z+1}{2}$.
5. Find the distance between the points P(6,5,9) and the plane determined by the points
6. A(3,-1,2),B(5,2,4) and C(-1,-1,6). (Ans: $\frac{6}{\sqrt{34}}$.)
7. Find the equation of the perpendicular drawn from the points (1,-2,3) to the plane 2x-3y+4z+9=0. Also, find the co-ordinates of the foot of the perpendicular. (Ans: -1,1,-1).
8. Find the equation of the planes through the points (2,-3,1) and (5,2,-1) and perpendicular to the planex-4y+5z+2=0.
9. Show that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y+1}{5} = z$ intersect. Find the point of intersection also.
10. Find the vector equation of the plane passing through the points A(2,2,-1), B(3,4,2) and C(7,0,6). Also find the Cartesian equation of the plane.
11. Show that the lines $\frac{x+3}{-3} = \frac{y-1}{1} = \frac{z-5}{5}$ and $\frac{x+1}{-1} = \frac{y-2}{2} = \frac{z-5}{5}$ are co-planer .Also find the equation of the plane containing the lines

PROBABILITY

Q1. A die is thrown twice and the sum of the numbers appearing is observed to be 7. What is the conditional probability that the number 4 has appeared at least once? (4)

Q2.Given that the two numbers appearing on throwing two dice are different. Find the probability of the event 'the sum of numbers on the dice is 6' (4)

Q3.Events A and B are such that P(a) = ½ , P(B) =7/12 and P(not A or not B)= ¼.State whether A and B are independent. (4)

Q4. Two balls are drawn at random with replacement from a box containing 8 black and 10 red balls. Find the probability that

(i) Both balls are black

(ii) First ball is red and second is black (4)

Q5. In answering a question on a multiple choice test, a student either knows the answer or guesses.

Let ¼ be the probability that he knows the answer and ¾ be the probability that he guesses. Assuming that , a student who guesses the answer will be correct with probability ¼.What is the probability that the student knows the answer given that he answered it correctly. (6)

Q6.There are three coins. One is a two headed coin(having head on both faces., another is a biased coin that comes up heads75% of the time and third is an unbiased coin. One of the three coins is chosen at random and tossed, it shows heads, what is the probability that it was the biased coin? (6)

Q7. Find the probability distribution of number of doublets in three throws of a pair of dice. (6)

Q8.Two cards are drawn simultaneously from a well shuffled pack of 52 cards. Find the mean, variance and standard deviation of the number of queens. (6)

Q9. Find the probability of throwing at least 2 sixes in 6 throws of a single die. (4)

Q10. How many times must a man toss a fair coin so that the probability of having at least one tail is more than 80%? (4)